

Section 4.4

Rules for combining and manipulating means

* X and Y are random variables


* a and b are fixed numbers

$\mu_X = \text{mean of r.v. } X$

$\sigma_X = \text{std. dev. of r.v. } X$

$\mu_{2X} = \text{mean of } 2 \times \text{r.v. } X$

Rule for multiplication and addition of one variable


$$\mu_{a+bX} = \mu_X \cdot b + a$$

Rule for combining X and Y together:

$$\mu_{X+Y} = \mu_X + \mu_Y$$

Example:

1) Random variable X has mean of 6.2. Find the mean if we divide by 12 and add 3.

$$\mu_X = 6.2$$

$$\mu_{\frac{1}{12}X+3} = \frac{1}{12}(\mu_X) + 3 = \frac{1}{12}(6.2) + 3 = 3.52$$

2) Random variable Y has a mean of 3.4. Find the mean of X + Y

$$\mu_Y = 3.4$$

$$\mu_{X+Y} = \mu_X + \mu_Y = 6.2 + 3.4 = 9.6$$

Rules for combining and manipulating std. deviations

Notes:

* Variance = $(\text{std. dev})^2 = \sigma^2$

* ALWAYS... work in σ^2
take $\sqrt{\quad}$ to get σ

Rule for multiplication and addition of one variable:

$$\sigma_{a+bX} = \sigma_{bX} \Rightarrow \sigma_{bX}^2 = b^2 \sigma_X^2$$



Rule for combining X and Y together:

$$\sigma_{X+Y} \Rightarrow \sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2 = \sqrt{\sigma_X^2 + \sigma_Y^2}$$



$$\sigma_{X+Y} = \sigma_{X-Y}$$

Example:

Random Variable X has std. dev = 3.1

Random Variable Y has std. dev = 1.4

$$\sigma_x = 3.1$$

$$\sigma_y = 1.4$$

Find:

$$\sigma_{x+y} \Rightarrow \sigma_{x+y}^2 = \sigma_x^2 + \sigma_y^2 = (3.1)^2 + (1.4)^2 \\ = \sqrt{11.57} = \textcircled{3.401}$$

$$\sigma_{x-y} = \textcircled{3.401}$$

$$\sigma_{2x} = \sigma_{2x} \Rightarrow \sigma_{2x}^2 = 2^2 \sigma_x^2 = 2^2 (3.1)^2 = \sqrt{38.44} \\ \downarrow \quad \quad \quad \downarrow \\ 2\sigma_x \quad \quad \quad \sqrt{2} \sigma_x \quad \quad \quad \textcircled{6.2}$$

Complete worksheet 4.4- Rules for means and variances.

$$\sigma_{3x+y-x} =$$

$$\sigma_{3x+y} \Rightarrow \sigma_{3x+y}^2 = \sigma_{3x}^2 + \sigma_y^2$$
$$3^2(3.1)^2 + (1.4)^2$$

$$\sqrt{88.45}$$

$$= 9.405$$

Answers:

- 1) a) 50 b) 7 c) 47 d) 10 e) 2
- 2) a) -5 b) 55 c) 3.606 d) 3.606 e) 11.662
- 3) mean of $M+V = 900$
std. dev. of $M+V = 141.42$
- 4) mean of $M+V+A = 1476$
std. dev. of $M+V+A = 172.203$

page 344-345 #70

a) mean $Y-X = 0.001$
std. dev. $Y-X = 0.0022$

b) mean $Z = 2.0005$
std. dev. $Z = 0.001118$

$$\sigma_{\frac{1}{2}X + \frac{1}{2}Y}$$